(N)MRI: A DEEPER LOOK

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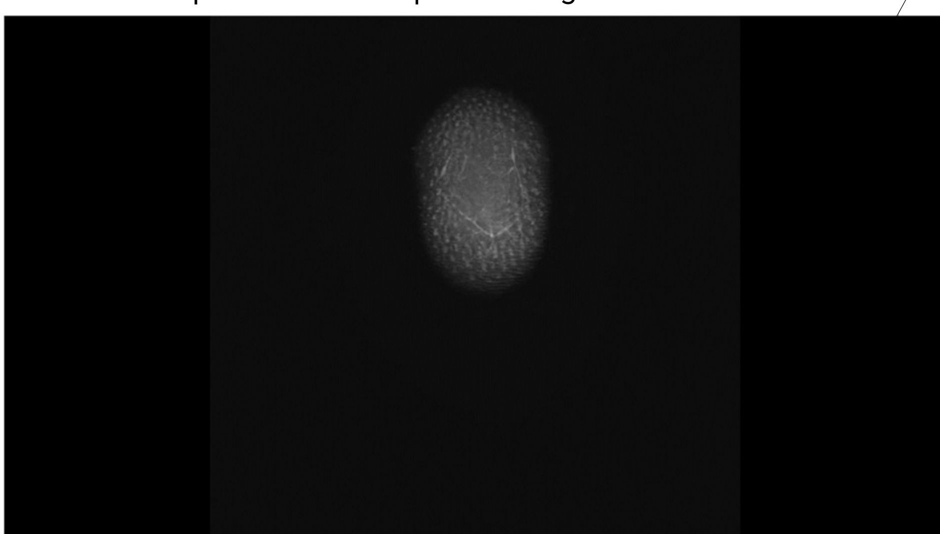
WHOSE KNEE IS THIS?

Extensive displaced and complex tearing of the lateral meniscus



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[1] History of MRI

[2] NMR for one particle / a bulk

[3] Building a machine

[4] Implications for medicine

AGENDA

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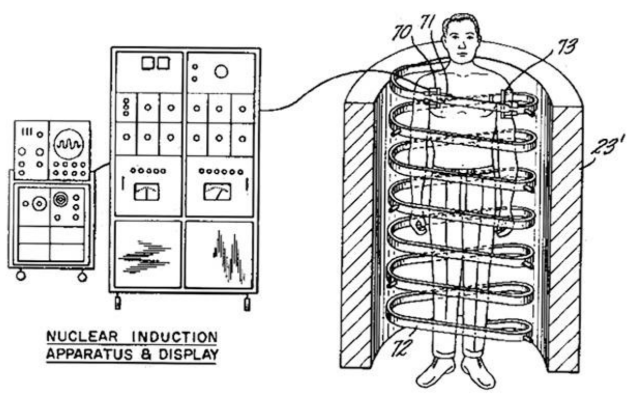
HISTORY OF MRI



Raymond Damadian’s 1969 "Apparatus and method for detecting cancer in tissue."

Damadian’s 1977 scan of Larry Minkoff

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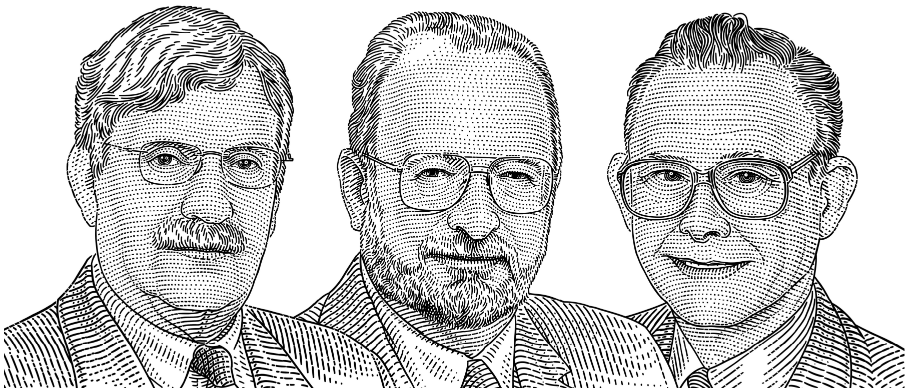
HISTORY OF MRI

Raymond Damadian | Paul Lauterbur | Peter Mansfield



Raymond Damadian:
- Original Inventor
- First prototype and successful scan

Paul Lauterbur:
- 2003 Nobel laureate
- Zeugmatography

Peter Mansfield:
- 2003 Nobel laureate
- Optimising scan process

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HISTORY OF MRI

Martin Wood.

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NMR FOR ONE PARTICLE

NUCLEAR MAGNETIC RESONANCE

[1] STERN-GERLACH

Inherent angular momentum

$\vec{\mu} = \gamma\hat{\vec{J}} \rightarrow \gamma\hat{\vec{S}}$

[2] HAMILTONIAN IN B-FIELD

Potential energy $U = -\vec{\mu} \cdot \vec{B}$

$\therefore \hat{H} = -\gamma\hat{\vec{S}} \cdot \vec{B} \rightarrow -\gamma B_z \widehat{S_z}$

[3] ZEEMAN SPLITTING

Quantization of z-spin

$E_{\pm} = \pm\frac{\hbar}{2}\gamma B_z$

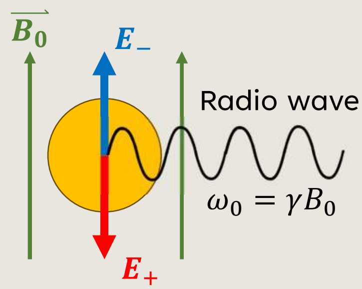
$\Delta E = \hbar\gamma B_z \ \ (= \hbar\omega)$

[4] NMR

Larmor frequency

$\overrightarrow{\boldsymbol{\omega_0}} = -\boldsymbol{\gamma}\overrightarrow{\boldsymbol{B_0}}$

Isidor Isaac Rabi, 1944 Nobel laureate



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NMR FOR A BULK

MAGNETISATION OF A VOXEL

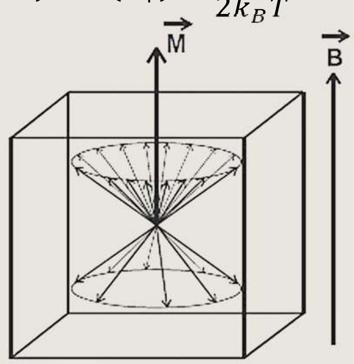
[1] CANONICAL ENSEMBLE

In voxel of uniform B-field

$P(E_{\pm}) = \exp(-\beta E_{\pm})/Z(\beta)$

With $\Delta E \ll k_B T$:

$P(E_-) - P(E_+) \approx \frac{\hbar\gamma B_0}{2k_B T}$

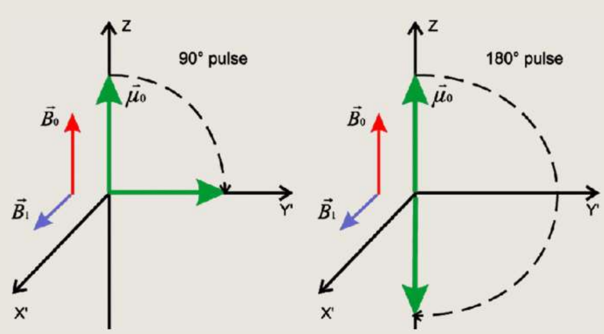


[2] NET MAGNETISATION

Using $\mu_z = \gamma\widehat{S_z}$:

$\overrightarrow{M_0} = n\frac{\gamma^2\hbar^2}{4k_B T}\overrightarrow{B_0}$ (H^1 density, n)

[3] NMR-EXCITATION



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NMR FOR A BULK
NUCLEAR RELAXATION

[4] LARMOR PRECESSION

Torque on dipole in uniform B-field

$\vec{\tau} = \dot{\vec{J}} = \vec{\mu} \times \overrightarrow{B_0}$

With $\gamma\vec{J} = \gamma\vec{S} = \vec{\mu}$ and $\overrightarrow{\omega_0} = -\gamma\overrightarrow{B_0}$:

$\dot{\vec{M}} = \overrightarrow{\omega_0} \times \vec{M}\ \ (\propto n\, e^{-i\omega_0 t})$

[5] BLOCH EQUATIONS

Energy emission gives Felix Bloch's equations (1944):

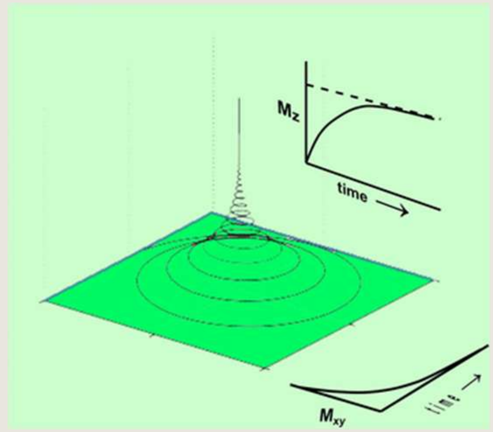
$$\dot{M}_z = \left(\overrightarrow{\omega_0} \times \vec{M}\right)_z - \frac{M_z - M_0}{T_1}$$

$$\dot{M}_{x,y} = \left(\overrightarrow{\omega_0} \times \vec{M}\right)_{x,y} - \frac{M_{x,y}}{T_2}$$

[5] RELAXATION TYPES

T1 (spin-lattice): energy dissipation

T2 (spin-spin): spins out of phase



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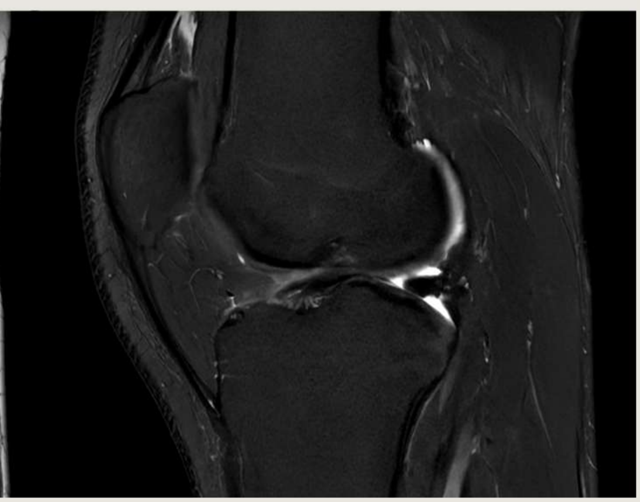
NMR FOR A BULK
RELAXATION CONTRAST

T1 contrasted side-view of my knee



T2 contrasted side-view of my knee

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BUILDING A MACHINE
STRUCTURAL DESIGN

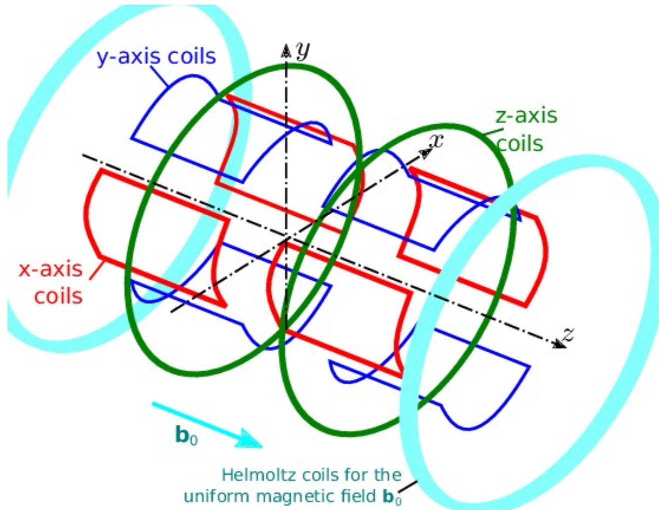
[1] DETECTING MAGNETISATION

Faraday's law of induction:

$emf = -\dot{\Phi} \propto \dot{M} \propto n\, e^{-i\omega_0 t}$

[2] SPATIAL LOCALISATION

1. Superconducting Helmholtz B-field
2. z-gradient coils isolate cross-section
3. x-gradient coils encode frequency
4. y-gradient coils encode phase (time T)



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BUILDING A MACHINE
FROM SIGNAL TO IMAGE

[3] VOXEL SIGNAL

Contribution from each voxel:
$dI(t) \propto n(x,y)\, dx\, dy\, e^{-i(\omega(x)t+\phi(y))}$

[4] CROSS-SECTION SIGNAL

Linear position dependence:

- $\omega(x) = \gamma(B_0 + b_x x)$
- $\phi(y) = \gamma b_y y T$

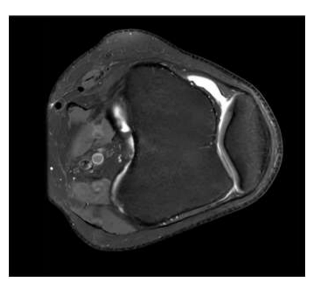
Overall signal from cross-section:
$I(t,T) \propto \int dx \int dy\, n(x,y)\, e^{-i\gamma(B_0+b_x x)t}\, e^{-i\gamma b_y y T}$

[5] FOURIER TRANSFORM

$\mathcal{F}^{(2)}[I](X,Y) \propto n\left(\frac{X-\gamma B_0}{\gamma b_x}, \frac{Y}{\gamma b_y}\right)$

Using $X = \gamma(b_x x + B_0)$ and $Y = \gamma b_y y$:
$\mathcal{F}^{(2)}[I]\big(\gamma(b_x x + B_0), \gamma b_y y\big) \propto n(x,y)$

This is an MRI cross-section image!



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IMPLICATIONS
FOR DIAGNOSTIC MEDICINE

[1] TRANSFORMATION OF DIAGNOSTIC MEDICINE

- Precisely target malignant tumors
- Locate target for heart stents
- **Makes operations more targeted**

[2] COMPARISON TO PRIOR METHODS

- Louder, takes longer, no metal implants
- Can image soft tissue with detail
- No harmful radiation (track pregnancy)
  - Compared to *barbaric* X-rays and CAT scans
- **MRI per capita measures national healthcare**

[3] FUTURE DEVELOPMENTS

- Larger bores for bariatrics and claustrophobia
- Cheaper refrigeration
- Smaller/lighter/cheaper machines
- Live radiotherapy & fMRI
- AI to diagnose conditions

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MEDIA AND QUOTES

[1] https://www.iec.ch/blog/invention-magnetic-resonance-imaging-mri

[2] https://affordablemri.com/who-invented-the-mri/

[3] https://humanprogress.org/heroes-of-progress-pt-47-damadian-lauterbur-and-mansfield/

[4] https://www.scielo.org.mx/scielo.php?script=sci_arttext&pid=S1870-35422017000100048#:~:text=MRI%20is%20a%20technique%20for,that%20help%20with%20medical%20diagnosis.

[5] https://www.researchgate.net/figure/Illustration-of-90-and-180-radiofrequency-pulses-in-the-rotating-frame_fig6_224837893

[6] https://mri-q.com/bloch-equations.html

[7] https://www.researchgate.net/figure/Representation-of-the-MRI-magnetic-gradient-coils-set-configuration-x-and-y-axes-are_fig4_312655674

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[12] https://www.youtube.com/watch?v=NlYXqRG7lus

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[16] https://www2.physics.ox.ac.uk/sites/default/files/2011-06-08/optics_notes_and_slides_part_5_pdf_63907.pdf

[17] https://royalsocietypublishing.org/doi/10.1098/rsbm.2023.0005

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APPENDIX FOR QUESTIONS

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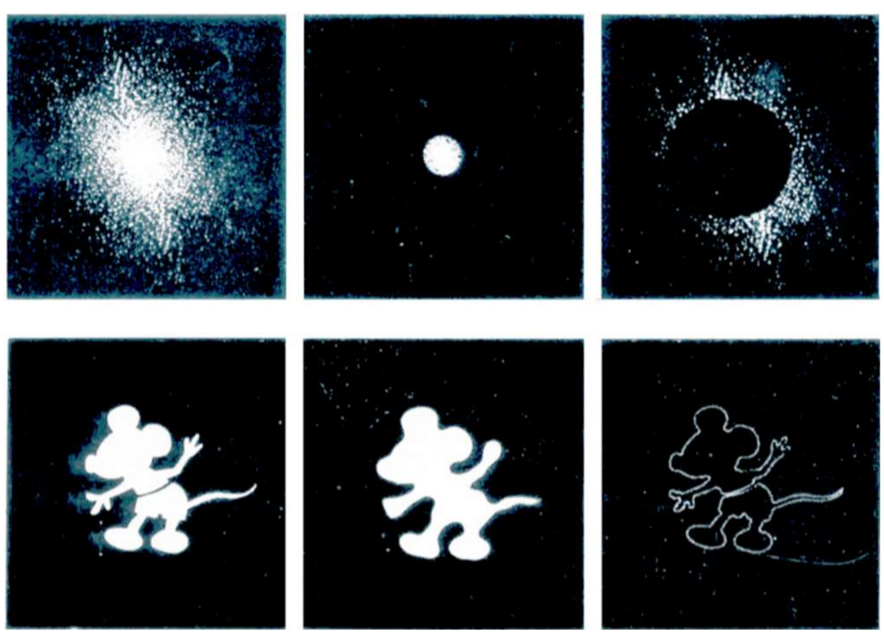
HIGH/LOW-PASS FILTERS



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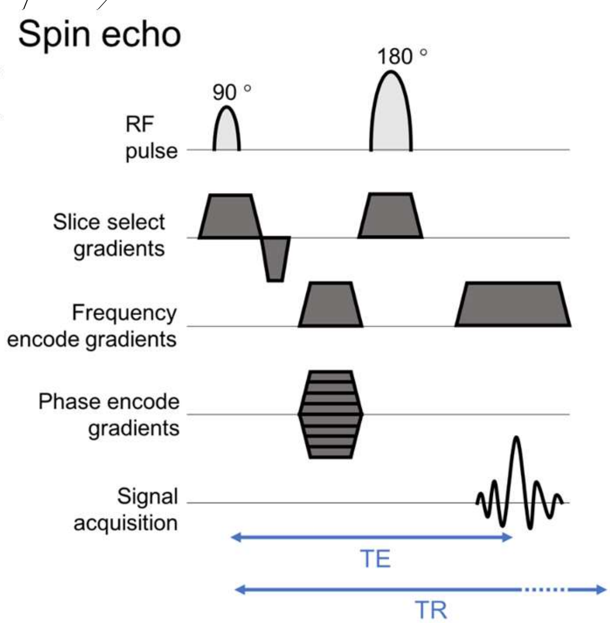
SPIN ECHO PULSE SEQUENCE



The current measured is always from $\dot{M}_{x,y}$.

TE: Time of echo

TR: Time of repetition

SCAN TYPE	TE	TR
Proton density	Short	Long
T1 weighted	Short	Short
T2 weighted	Long	Long

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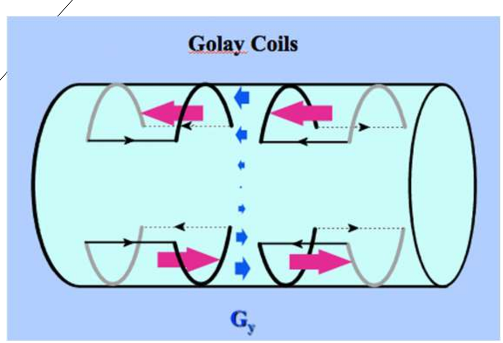
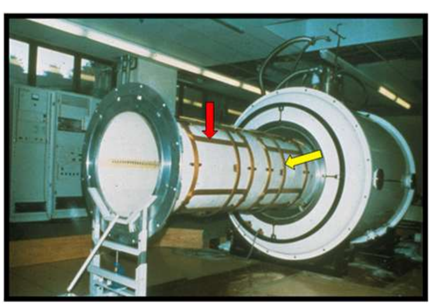
GOLAY COILS (X,Y GRADIENT FIELDS)





"Double saddle" coil configuration,

Marcus Golay, 1958

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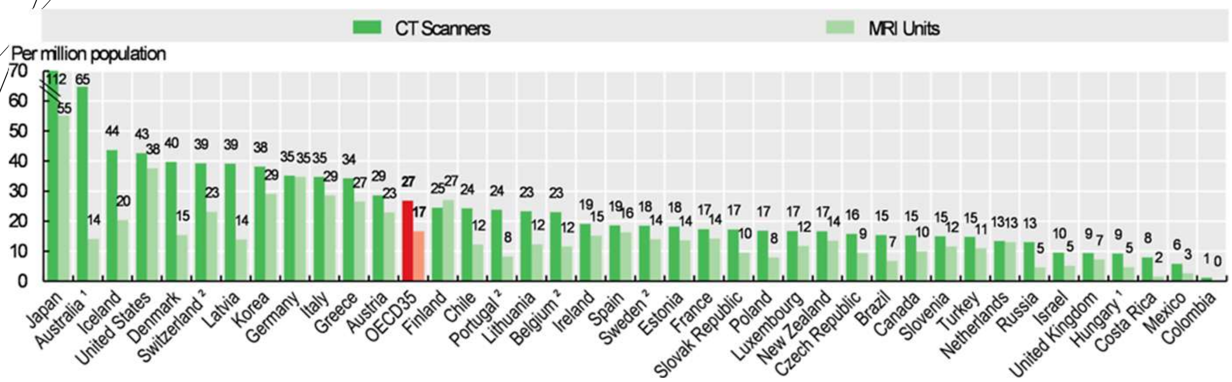
MRI UNITS PER CAPITA



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INDUCED CURRENT FROM MAGNETISED SAMPLE

$$emf = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$\vec{B} = \vec{\nabla}\times\vec{A}$ with:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3\vec{r}'$$

We also have:

$J(\vec{r},t) = \vec{\nabla}\times\vec{M}(\vec{r},t)$

Substitute it all in, integrate by parts and we get:

$$emf(\mathrm{t}) = -\frac{d}{dt}\int \vec{B}^r(\vec{r})\cdot\vec{M}(\vec{r},t)\; d^3\vec{r}$$

where $\vec{B}^r(\vec{r})$ is the magnetic field that would be produced at position $\vec{r}$, per unit current, were the coil being used to create a B-field.

The integral is taken over the sample.

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